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New gauge bosons from the littlest Higgs model and the process $e^+e^- \to t\bar{t}$

Chong-Xing Yue[1,a], Lei Wang, Jian-Xing Chen

Department of Physics, Liaoning Normal University, Dalian 116029, P.R. China

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Abstract. In the context of the littlest Higgs (LH) model, we study the process $e^+e^- \to t\bar{t}$. We find that the new gauge bosons Z_H and B_H can produce significant correction effects on this process, which can be further enhanced by suitably polarized beams. In most of the parameter space preferred by the electroweak precision data, the absolute value of the relative correction parameter R_{B_H} is larger than 5%. As long as $1\,\mathrm{TeV} \leq M_{Z_H} \leq 1.5\,\mathrm{TeV}$ and $0.3 \leq c \leq 0.5$, the absolute value of the relative correction parameter R_{Z_H} is larger than 5%. With reasonable values of the parameters of the LH model, the possible signals of the new gauge bosons B_H and Z_H can be detected via the process $e^+e^- \to t\bar{t}$ in the future LC experiments with the CM energy $\sqrt{S} = 800\,\mathrm{GeV}$. B_H exchange and Z_H exchange can generate significantly corrections to the forward–backward asymmetry $A_{\mathrm{FB}}(t\bar{t})$ only in a small part of the parameter space.

1 Introduction

Although the standard model (SM), based on the gauge group $SU(2)_{\mathrm{L}} \times U(1)_Y$, has been successful in describing the physics of electroweak interactions, the mechanism of electroweak symmetry breaking (EWSB) and the origins of the masses of the elementary fermions are still unknown. Furthermore, its scalar sector suffers from the problems of triviality and unnaturalness, etc. Thus, it is quite possible that the SM is only an effective theory valid below some high energy scale. New physics (NP) should exist at energy scales around TeV.

Recently, a kind of theory for EWSB was proposed to solve the hierarchy between the TeV scale of possible NP and the electroweak scale $v = 246\,\mathrm{GeV}$, which is known as "little Higgs models" [1–3]. The key feature of these models is that the Higgs boson is a pseudo-Goldstone boson of a global symmetry which is spontaneously broken at some higher scale f and thus is naturally light. EWSB is induced by a Coleman–Weinberg potential, which is generated by integrating out the heavy degrees of freedom. This type of models can be regarded as one of the important candidates of the NP beyond the SM.

A high energy e^+e^- linear collider (LC) will offer an opportunity to make precision measurements of the properties of the electroweak gauge bosons, top quarks, Higgs bosons and also to constrain NP [4]. In the LC experiments, top quark pairs are mainly produced from the S-channel exchange of the SM gauge bosons γ and Z via the process $e^+e^- \to t\bar{t}$ [5]. The total cross section is of the order of 1 pb, so that top quark pairs will be produced at large rates in a clean environment at LC. If we assume that the integrated luminosity $\mathcal{L}_{\mathrm{int}}$ is about $100\,\mathrm{fb}^{-1}$, there will be several times 10^4 top quark pairs to be generated in the future LC experiments. Furthermore, the QCD and EW corrections to the process $e^+e^- \to t\bar{t}$ are small and decrease with the center-of-mass (CM) energy $\sqrt{S}$ increasing. The option of longitudinally polarized beams can help to improve the measurement precision and reduce the background in search for NP. Thus, theoretical calculations of new particle contributions to the process $e^+e^- \to t\bar{t}$ are of much interest for testing of NP.

In general, the new gauge bosons are heavier than the current experimental limits on direct searches. However, these new particles may produce virtual effects on some physical observables, which may be detected in the present or future high energy experiments. In [6], we discussed the possibility of detecting the new gauge bosons Z_H and B_H predicted by the littlest Higgs (LH) model [1] in the future LC experiments with CM energy $\sqrt{S} = 500\,\mathrm{GeV}$ and integrating luminosity $\mathcal{L}_{\mathrm{int}} = 340\,\mathrm{fb}^{-1}$ and both beams polarized via considering their contributions to the processes $e^+e^- \to f\bar{f}$ with $f = \tau, \mu, b$ and c. Since the masses of these fermions are largely smaller than the CM energy $\sqrt{S}$, we have neglected the masses of these fermions in our numerical estimations. Our results show that the new gauge bosons Z_H and B_H can indeed produce significant contributions to these processes in most of the parameter space preferred by the electroweak precision data, which might be observable in future LC experiments. The aim of this paper is to consider the contributions of the Z_H and B_H to the process $e^+e^- \to t\bar{t}$ and discuss whether these new particles can be detected via this process in future LC

[a] e-mail: cxyue@lnnu.edu.cn

experiments with CM energy $\sqrt{S} = 800\,\text{GeV}$ and integrating luminosity $\mathcal{L}_{\text{int}} = 580\,\text{fb}^{-1}$. We find that the absolute value of the relative correction parameter R_{B_H} generated by B_H exchange is larger than 8% in most of the parameter space of the LH model preferred by the electroweak precision data. As long as $1\,\text{TeV} \leq M_{Z_H} \leq 1.5\,\text{TeV}$ and $0.3 \leq c \leq 0.5$, the absolute value of R_{Z_H} is larger than 5%. If we assume that the initial electron and positron beams are suitably polarized, the absolute values of the relative correction parameters R_{B_H} and R_{Z_H} can be enhanced. Thus, with reasonable values of the parameters of the LH model, the possible signals of the new gauge bosons B_H and Z_H can be detected in future LC experiments with the CM energy $\sqrt{S} = 800\,\text{GeV}$, which is similar to the conclusions given in [6]. We further calculate the contributions of these new gauge bosons to the forward–backward asymmetry $A_{\text{FB}}(t\bar{t})$. We find that they can generate significant corrections to the forward–backward asymmetry $A_{\text{FB}}(t\bar{t})$ only in a small part of the parameter space.

In Sect. 2, we give the formula of the contributions of the new gauge bosons B_H and Z_H to the process $e^+e^- \to t\bar{t}$ and estimate the values of the relative corrections parameters $R_{B_H} = \sigma^{B_H}(t\bar{t})/\sigma^{\text{SM}}(t\bar{t})$ and $R_{Z_H} = \sigma^{Z_H}(t\bar{t})/\sigma^{\text{SM}}(t\bar{t})$. The dependence of the relative correction parameters R_{B_H} and R_{Z_H} on the initial beam polarization is discussed in Sect. 3. In Sect. 4, we calculate the contributions of these new gauge bosons to the forward–backward asymmetry $A_{\text{FB}}(t\bar{t})$. Our conclusions and discussions are given in Sect. 5.

2 Corrections of the new gauge bosons B_H and Z_H to the process $e^+e^- \to t\bar{t}$

The LH model [1] is one of the simplest and phenomenologically viable models, which realizes the little Higgs idea. It consists of a non-linear σ model with a global $SU(5)$ symmetry, which is broken down to its subgroup $SO(5)$ by a vacuum condensate $f \sim \Lambda s/4\pi \sim \text{TeV}$. At the same time, the locally gauged group $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ is broken to its diagonal subgroup $SU(2) \times U(1)$, identified as the SM electroweak gauge group. This breaking scenario gives rise to four massive gauge bosons B_H, Z_H, and $W_H^{\pm}$, which might produce characteristic signatures at the present and future high energy collider experiments [7–9].

Taking account of the gauge invariance of the Yukawa coupling and the $U(1)$ anomaly cancellation, the coupling expressions of the gauge bosons B_H and Z_H to ordinary particles, which are related our calculation, can be written as [7]

$$g_V^{B_H ee} = \frac{3e}{4C_w s' c'}\left(c'^2 - \frac{2}{5}\right),$$

$$g_A^{B_H ee} = \frac{e}{4C_w s' c'}\left(c'^2 - \frac{2}{5}\right); \tag{1}$$

$$g_V^{B_H tt} = \frac{e}{2C_w s' c'}\left[\frac{5}{6}\left(\frac{2}{5} - c'^2\right) - \frac{1}{5}x_{\text{L}}\right],$$

$$g_A^{B_H tt} = \frac{e}{2C_w s' c'}\left[\frac{1}{2}\left(\frac{2}{5} - c'^2\right) - \frac{1}{5}x_{\text{L}}\right]; \tag{2}$$

$$g_V^{Z_H ee} = -\frac{ec}{4S_w s}, \quad g_A^{Z_H ee} = \frac{ec}{4S_w s}; \tag{3}$$

$$g_V^{Z_H tt} = \frac{ec}{4S_w s}, \quad g_A^{Z_H tt} = -\frac{ec}{4S_w s}, \tag{4}$$

where $S_{\text{W}} = \sin\theta_{\text{W}}, \theta_{\text{W}}$ is the Weinberg angle. Using the mixing parameters c ($s = \sqrt{1-c^2}$) and c' ($s' = \sqrt{1-c'^2}$), we can represent the SM gauge coupling constants as $g = g_1 s = g_2 c$ and $g' = g_1' s' = g_2' c'$. The mixing angle parameter between the SM top quark t and the vector-like quark T is defined as $x_{\text{L}} = \lambda_1^2/(\lambda_1^2 + \lambda_2^2)$, in which λ_1 and λ_2 are the Yukawa coupling parameters.

Global fits to the eletroweak precision data produce rather severe constraints on the parameter space of the LH model [10]. However, if the SM fermions are charged under $U(1)_1 \times U(1)_2$, the constraints become relaxed. The scale parameter $f = 1 \sim 2\,\text{TeV}$ is allowed for the mixing parameters c, c', and x_{L} in the ranges of $0 \sim 0.5$, $0.62 \sim 0.73$, and $0.3 \sim 0.6$, respectively [11]. In this case, the masses of B_H and Z_H are allowed in the ranges of $300\,\text{GeV} \sim 900\,\text{GeV}$ and $1\,\text{TeV} \sim 3\,\text{TeV}$, respectively. Thus, we will take the Z_H mass M_{Z_H}, B_H mass M_{B_H} and the mixing parameters c, c' and x_{L} as free parameters in our calculation.

For the SM, top quark pair $t\bar{t}$ can be produced in sufficient abundance in the LC experiments. The main production mechanism proceed at the Born level by the S-channel annihilation of an initial electron–positron pair into virtual photon or neutral gauge boson Z, and their subsequent splitting into top quark pairs, $e^+e^- \to \gamma, Z \to t\bar{t}$. For the LH model, the B_H exchange and Z_H exchange can also produce top quark pairs. The production cross sections can be written as

$$\begin{aligned}
\sigma^{B_H}(t\bar{t}) &= \frac{N_c^f \beta}{8\pi S} \\
&\times \Bigg\{ \left(1 - \frac{\beta^2}{3}\right)\frac{4}{3}e^2 g_V^{B_H ee} g_V^{B_H tt} \frac{S\left(M_{B_H}^2 - S\right)}{\left(S - M_{B_H}^2\right)^2 + M_{B_H}^2 \Gamma_{B_H}^2} \\
&+ \left[\left(g_V^{B_H ee}\right)^2 + \left(g_A^{B_H ee}\right)^2\right] \\
&\times \Bigg[\left(1 - \frac{\beta^2}{3}\right)\left[\left(g_V^{B_H tt}\right)^2 + \left(g_A^{B_H tt}\right)^2\right] \\
&\quad - \left(1-\beta^2\right)\left(g_A^{B_H tt}\right)^2\Bigg] \frac{S^2}{\left(S - M_{B_H}^2\right)^2 + M_{B_H}^2 \Gamma_{B_H}^2} \\
&+ \left(g_V^{Zee} g_V^{B_H ee} + g_A^{Zee} g_A^{B_H ee}\right) \\
&\times \left[\left(1 - \frac{\beta^2}{3}\right)\left(g_V^{Ztt} g_V^{B_H tt} + g_A^{Ztt} g_A^{B_H tt}\right)\right. \\
&\quad \left. - \left(1-\beta^2\right)\left(g_A^{B_H tt}\right)\left(g_A^{Ztt}\right)\right] \\
&\times \frac{2S^2\left[\left(S - M_Z^2\right)\left(S - M_{B_H}^2\right) + M_Z \Gamma_Z M_{B_H} \Gamma_{B_H}\right]}{\left[\left(S - M_Z^2\right)^2 + M_Z^2 \Gamma_Z^2\right]\left[\left(S - M_{B_H}^2\right)^2 + M_{B_H}^2 \Gamma_{B_H}^2\right]} \Bigg\},
\end{aligned} \tag{5}$$

$$\sigma^{Z_H}(t\bar{t}) = \frac{N_c^f \beta}{8\pi S}$$
$$\times \left\{ \left(1-\frac{\beta^2}{3}\right) \frac{4}{3} e^2 g_V^{Z_H ee} g_V^{Z_H tt} \frac{S\left(M_{Z_H}^2 - S\right)}{\left(S-M_{Z_H}^2\right)^2 + M_{Z_H}^2 \Gamma_{Z_H}^2} \right.$$
$$+ \left[\left(g_V^{Z_H ee}\right)^2 + \left(g_A^{Z_H ee}\right)^2 \right]$$
$$\times \left[\left(1-\frac{\beta^2}{3}\right) \left[\left(g_V^{Z_H tt}\right)^2 + \left(g_A^{Z_H tt}\right)^2 \right] \right.$$
$$\left. - \left(1-\beta^2\right) \left(g_A^{Z_H tt}\right)^2 \right]$$
$$\times \frac{S^2}{\left(S-M_{Z_H}^2\right)^2 + M_{Z_H}^2 \Gamma_{Z_H}^2}$$
$$+ \left(g_V^{Zee} g_V^{Z_H ee} + g_A^{Zee} g_A^{Z_H ee}\right)$$
$$\times \left[\left(1-\frac{\beta^2}{3}\right) \left(g_V^{Ztt} g_V^{Z_H tt} + g_A^{Ztt} g_A^{Z_H tt}\right) \right.$$
$$\left. - \left(1-\beta^2\right) \left(g_A^{Z_H tt}\right) \left(g_A^{Ztt}\right) \right] \tag{6}$$
$$\left. \times \frac{2S^2 \left[\left(S-M_Z^2\right)\left(S-M_{Z_H}^2\right) + M_Z \Gamma_Z M_{Z_H} \Gamma_{Z_H}\right]}{\left[\left(S-M_Z^2\right)^2 + M_Z^2 \Gamma_Z^2\right] \left[\left(S-M_{Z_H}^2\right)^2 + M_{Z_H}^2 \Gamma_{Z_H}^2\right]} \right\},$$

with

$$g_V^{Zee} = \frac{e}{4S_W C_W}\left(-1+4S_W^2\right), \quad g_A^{Zee} = \frac{e}{4S_W C_W}, \tag{7}$$
$$g_V^{Ztt} = \frac{e}{4S_W C_W}\left(1-\frac{8}{3}S_W^2\right), \quad g_A^{Ztt} = \frac{e}{4S_W C_W}, \tag{8}$$

where $\beta = \sqrt{1-\frac{4m_t^2}{S}}$, m_t is the top quark mass. The Γ_i represent the total decay widths of the gauge bosons Z, Z_H, and B_H. Γ_{Z_H} and Γ_{B_H} have been given in [6]. From the above equations, we can see that $\sigma^{B_H}(t\bar{t})$ mainly depends on the free parameters M_{B_H}, c' and x_L, while $\sigma^{Z_H}(t\bar{t})$ only depends on the free parameters c and M_{Z_H}, which is different from those for the process $e^+e^- \to f\bar{f}$ with $f = \tau, \mu, b$ and c. In that case, the contributions of the gauge bosons B_H are independent of the mixing parameter x_L. Thus, in this paper, we will take the mixing parameters c, c' and x_L as free parameters. Certainly, due to mixing between the gauge bosons Z and Z_H, the SM tree-level couplings $Ze\bar{e}$ and $Zt\bar{t}$ receive corrections at the order of v^2/f^2, which can also produce contributions to the production cross section of the process $e^+e^- \to t\bar{t}$. However, the contributions are suppressed by the factor v^4/f^4, which are smaller than those of B_H or Z_H. Thus, we have neglected this kind of corrections in the above equations.

To see the correction effects of B_H exchange and Z_H exchange on the $t\bar{t}$ production cross section, we plot the relative correction parameters $R_{B_H} = \sigma^{B_H}(t\bar{t})/\sigma^{SM}(t\bar{t})$ and $R_{Z_H} = \sigma^{Z_H}(t\bar{t})/\sigma^{SM}(t\bar{t})$ as functions of M_{B_H} and M_{Z_H} in Figs. 1 and 2, respectively. From these figures, we can see that the gauge boson Z_H decreases the SM $t\bar{t}$ production cross section $\sigma^{SM}(t\bar{t})$ in all of the parameter space, which satisfies the electroweak precision constraints. In most of the parameter space, the absolute value of the relative correction parameter R_{Z_H} is smaller than 5%, which is very difficult to be detected in the future LC experiments. This is consistent with the contributions of Z_H to the process $e^+e^- \to f\bar{f}$, which have been studied in [6]. However, for the gauge boson B_H, this is not the case. For $M_{B_H} \le 800$ GeV, B_H exchange produce positive corrections to the $t\bar{t}$ production cross section $\sigma^{SM}(t\bar{t})$ and the value of R_{B_H} increase as M_{B_H}, x_L and c' increasing. For 800 GeV $< M_{B_H} \le 900$ GeV, B_H exchange decreases the cross section $\sigma^{SM}(t\bar{t})$ and the absolute of R_{B_H} increase as M_{B_H} decreasing and x_L, c' increasing. The peak of the R_{B_H} resonance emerges when the B_H mass M_{B_H} is approximately equal to the CM energy $\sqrt{S} = 800$ GeV. In most of the parameter space, the absolute value of R_{B_H} is larger than 8%. Thus, the virtual effects of B_H on the process $e^+e^- \to t\bar{t}$ should be easy detected in the future LC experiments with $\sqrt{S} = 800$ GeV and $\mathcal{L}_{\text{int}} = 580\,\text{fb}^{-1}$.

3 The dependence of the relative correction parameters R_{B_H} and R_{Z_H} on the electron and positron beam polarization

An LC has a large potential for the discovery of new particles and is well suited for a precise analysis of NP beyond the SM. At present, the existing proposals are designed with high luminosity of about $\mathcal{L}_{\text{int}} = 340\,\text{fb}^{-1}$ at $\sqrt{S} = 500$ GeV and $\mathcal{L}_{\text{int}} = 580\,\text{fb}^{-1}$ at $\sqrt{S} = 800$ GeV [4]. An important tool of an LC is the use of polarized beams. Beam polarization is not only useful for a possible reduction of the background, but might also serve as a possible tool to disentangle different contributions to the signal and lead to substantial enhancement of the produce cross sections of some processes [12]. To see whether the contributions of the new gauge bosons B_H and Z_H to the process $e^+e^- \to t\bar{t}$ can indeed be detected, we discuss the dependence of the relative correction parameters R_{B_H} and R_{Z_H} on the initial electron and positron beam polarization in this section.

Considering the polarization of the initial electron and positron beams, the cross section of the process $e^+e^- \to t\bar{t}$ can be generally written as

$$\sigma(t\bar{t}) = (1+P_e)(1-P_{\bar{e}})(\sigma_{RR}(t\bar{t}) + \sigma_{RL}(t\bar{t}))$$
$$+ (1-P_e)(1+P_{\bar{e}})(\sigma_{LL}(t\bar{t}) + \sigma_{LR}(t\bar{t})), \tag{9}$$

where P_e and $P_{\bar{e}}$ are the degrees of longitudinal electron and position polarization, respectively. The σ_{ij} are the chiral cross sections of this process. The relative correction parameters R_{B_H} and R_{Z_H} are plotted as functions of M_{B_H} and M_{Z_H} for $c' = 0.65$, $x_L = 0.5$, $c = 0.3$ and different beam polarizations in Figs. 3 and 4, respectively. In these two figures, we have used the solid line, dashed line, and the dotted line to represent $(P_e, P_{\bar{e}}) = (0, 0)$, $(0.8, -0.6)$, and $(-0.8, 0.6)$, respectively. Our calculation results show that the absolute values of $R_{B_H}[R_{Z_H}]$ for

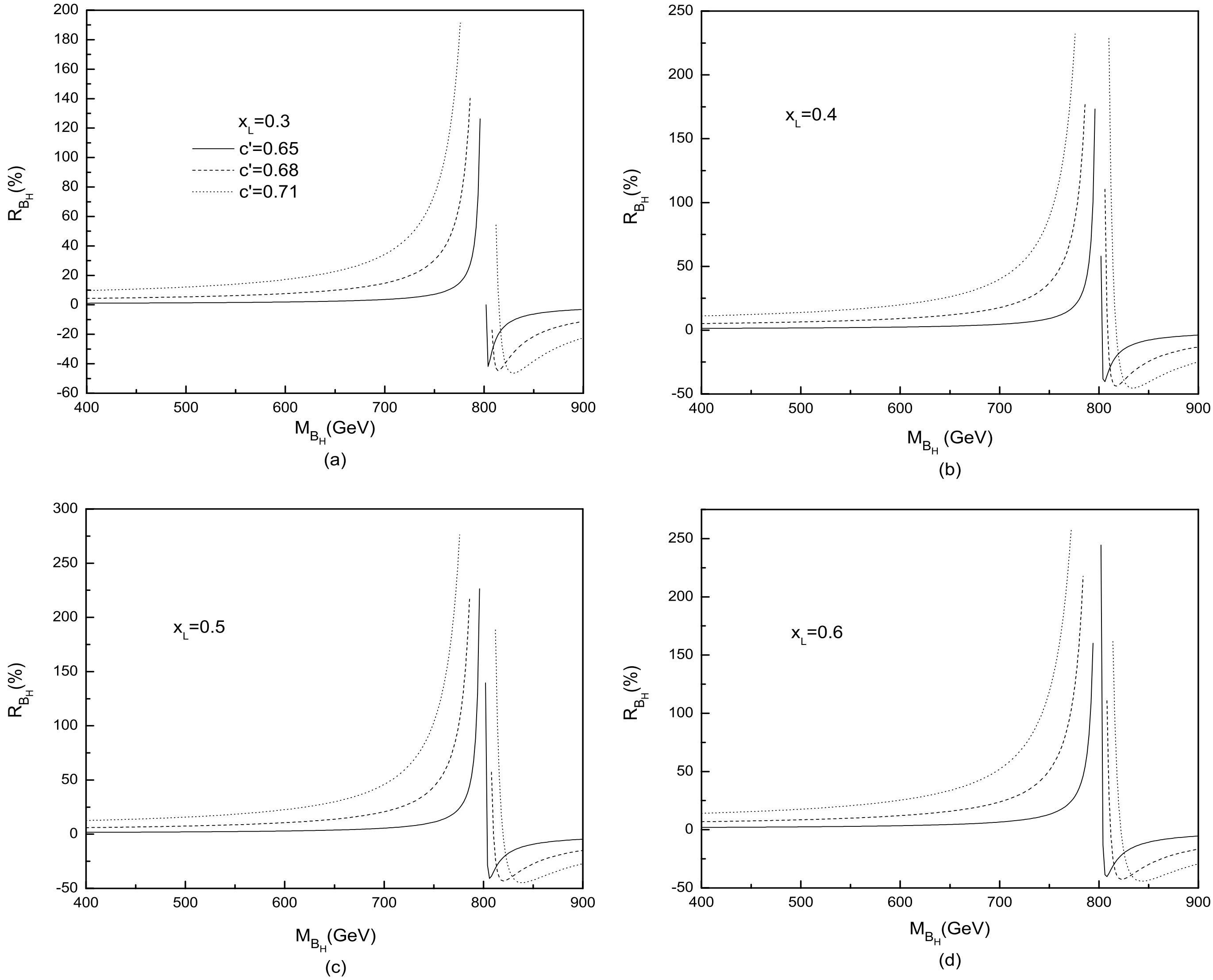

Fig. 1a–d. The relative correction parameter R_{B_H} as a function of the B_H mass M_{B_H} for different values of the mixing parameters c' and x_{L}

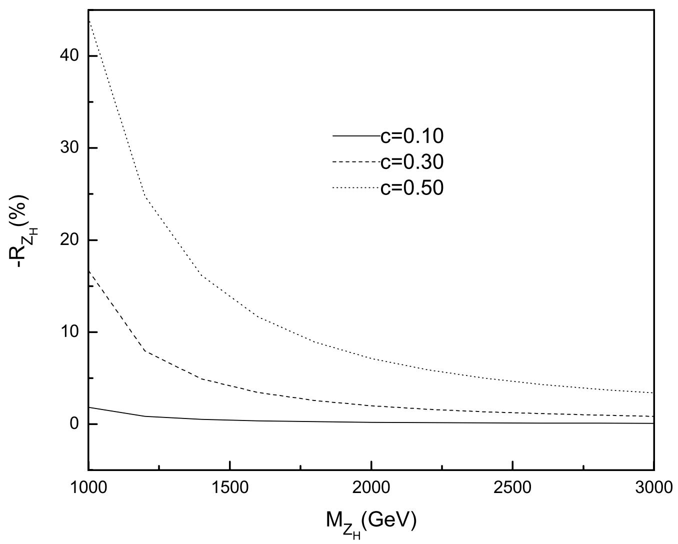

Fig. 2. The relative correction parameter R_{Z_H} as a function of the Z_H mass M_{Z_H} for three values of the mixing parameter c

$(P_e, P_{\bar{e}}) = (0.8, 0.6)[(-0.8, -0.6)]$ are smaller than those for $(P_e, P_{\bar{e}}) = (0, 0)$. Thus, in Figs. 3 and 4 we do not plot these lines.

From Figs. 3 and 4 we can see that the suitably polarized beams can indeed enhance the virtual effects of the new gauge bosons B_H and Z_H on the process $e^+e^- \to t\bar{t}$. In the whole parameter space preferred by the electroweak precision data, the value of R_{B_H} for $(P_e, P_{\bar{e}}) = (0.8, -0.6)$ is larger than that for $(P_e, P_{\bar{e}}) = (0, 0)$, while the absolute values of R_{Z_H} for $(P_e, P_{\bar{e}}) = (-0.8, 0.6)$ is larger than that for $(P_e, P_{\bar{e}}) = (0, 0)$. Varying the values of the free parameters c', x_{L}, and c does not change this conclusion. So, in Figs. 3 and 4 we have taken these parameters for fixed values $x_{\mathrm{L}} = 0.5$, $c' = 0.65$, and $c = 0.3$. Certainly, the values of R_{B_H} and R_{Z_H} change as the values of these parameters are varying. For example, for $0.3 \le c \le 0.5$ and $1\,\mathrm{TeV} \le M_{Z_H} \le 2\,\mathrm{TeV}$, the absolute value of R_{Z_H} for $(P_e, P_{\bar{e}}) = (-0.8, 0.6)$ is larger than 6%. The absolute value of R_{B_H} for $(P_e, P_{\bar{e}}) = (0.8, -0.6)$ is larger than 5% for $x_{\mathrm{L}} =$

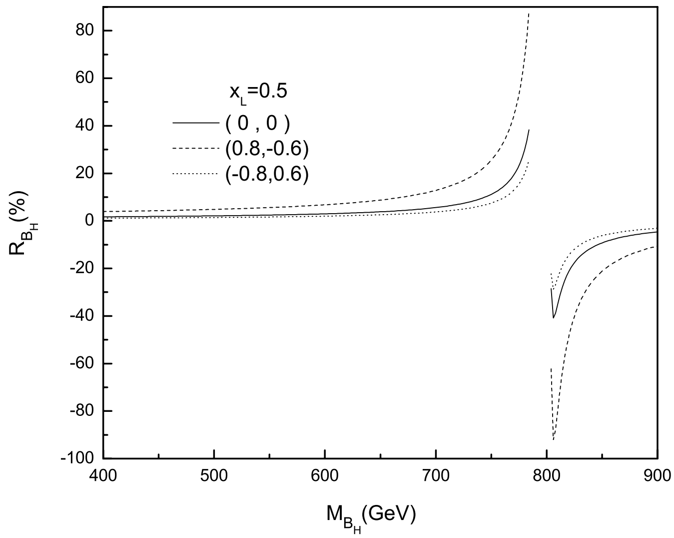

Fig. 3. The relative correction parameter R_{B_H} as a function of the B_H mass M_{B_H} for $c' = 0.65$, $x_{\mathrm{L}} = 0.5$, and $(P_e, P_{\bar{e}}) = (0,0), (0.8, -0.6), (-0.8, 0.6)$

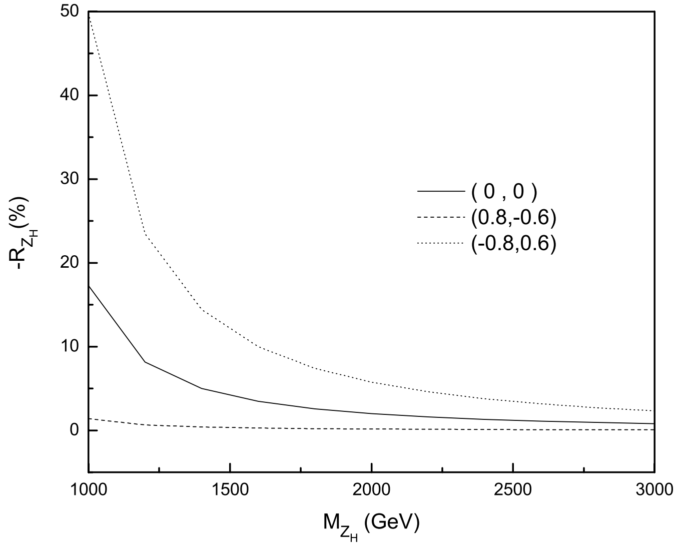

Fig. 4. The relative correction parameter R_{Z_H} as a function of the Z_H mass M_{Z_H} for $c = 0.3$ and $(P_e, P_{\bar{e}}) = (0,0), (0.8, -0.6), (-0.8, 0.6)$

0.5, $0.68 \le c' \le 0.73$ and $500\,\mathrm{GeV} < M_{B_H} \le 900\,\mathrm{GeV}$, but for $x_{\mathrm{L}} = 0.6$ its value is larger than 5% for $0.65 \le c' \le 0.73$ and $450\,\mathrm{GeV} \le M_{B_H} \le 900\,\mathrm{GeV}$. Thus, using the suitable polarization of the initial electron and positron beams, it is easier to detect the possible signals of the new gauge bosons B_H and Z_H in future LC experiments.

4 Gauge bosons B_H, Z_H and the forward–backward asymmetry $A_{\mathrm{FB}}(t\bar{t})$

The events generated by the process $e^+e^- \to f\bar{f}$ can be characterized by the momentum direction of the emitted fermion. If we assume that the final state fermion travels forward (F) or backward (B) with respect to the electron beam, than the forward–backward asymmetry can be defined by

$$A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}, \tag{10}$$

which is easier to be measured because only the identification of the charge of the fermion and the measurement of its direction are needed [13]. It can be measured for all tagged flavors and inclusively for hadrons. Thus, it is needed to calculate the contributions of B_H exchange and Z_H exchange to the forward–backward asymmetry $A_{\mathrm{FB}}(t\bar{t})$.

The total formula of $A_{\mathrm{FB}}(t\bar{t})$ for the new gauge bosons B_H and Z_H including the contributions of the SM gauge bosons γ and Z can be written as

$$A_{\mathrm{FB}}^{B_H}(t\bar{t}) = \frac{M_2^{B_H}(t\bar{t})}{M_1^{B_H}(t\bar{t})}, \qquad A_{\mathrm{FB}}^{Z_H}(t\bar{t}) = \frac{M_2^{Z_H}(t\bar{t})}{M_1^{Z_H}(t\bar{t})}, \tag{11}$$

where

$$\begin{aligned} & M_2^{B_H}(t\bar{t}) \\ &= \beta \left\{ \frac{2e^2}{3} g_A^{Zee} g_A^{Ztt} \frac{4S\left(M_Z^2 - S\right)}{\left(S - M_Z^2\right)^2 + M_Z^2 \Gamma_Z^2} \right. \\ &\quad + \frac{2e^2}{3} g_A^{B_H ee} g_A^{B_H tt} \frac{4S\left(M_{B_H}^2 - S\right)}{\left(S - M_{B_H}^2\right)^2 + M_{B_H}^2 \Gamma_{B_H}^2} \\ &\quad + g_V^{Zee} g_A^{Zee} g_V^{Ztt} g_A^{Ztt} \frac{8S^2}{\left(S - M_Z^2\right)^2 + M_Z^2 \Gamma_Z^2} \\ &\quad + g_V^{B_H ee} g_A^{B_H ee} g_V^{B_H tt} g_A^{B_H tt} \frac{8S^2}{\left(S - M_{B_H}^2\right)^2 + M_{B_H}^2 \Gamma_{B_H}^2} \\ &\quad + \left(g_V^{Zee} g_A^{B_H ee} + g_V^{B_H ee} g_A^{Zee}\right)\left(g_V^{Ztt} g_A^{B_H tt} + g_A^{Ztt} g_V^{B_H tt}\right) \\ &\quad \left. \times \frac{4S^2\left[\left(S - M_Z^2\right)\left(S - M_{B_H}^2\right) + M_Z \Gamma_Z M_{B_H} \Gamma_{B_H}\right]}{\left[\left(S - M_Z^2\right)^2 + M_Z^2 \Gamma_Z^2\right]\left[\left(S - M_{B_H}^2\right)^2 + M_{B_H}^2 \Gamma_{B_H}^2\right]} \right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} & M_1^{B_H}(t\bar{t}) \\ &= \left\{ \frac{16e^4}{9}\left(1 - \frac{\beta^2}{3}\right) \right. \\ &\quad + \frac{8e^2}{3}\left(1 - \frac{\beta^2}{3}\right) g_V^{Zee} g_V^{Ztt} \frac{2S\left(M_Z^2 - S\right)}{\left(S - M_Z^2\right)^2 + M_Z^2 \Gamma_Z^2} \\ &\quad \times \left[\left(g_V^{Zee}\right)^2 + \left(g_A^{Zee}\right)^2\right] \\ &\quad \times \left[4\left(1 - \frac{\beta^2}{3}\right)\left[\left(g_V^{Ztt}\right)^2 + \left(g_A^{Ztt}\right)^2\right] - 4\left(1 - \beta^2\right)\left(g_A^{Ztt}\right)^2\right] \\ &\quad \times \frac{S^2}{\left(S - M_Z^2\right)^2 + M_Z^2 \Gamma_Z^2} \\ &\quad + \frac{8e^2}{3}\left(1 - \frac{\beta^2}{3}\right) g_V^{B_H ee} g_V^{B_H tt} \frac{2S\left(M_{B_H}^2 - S\right)}{\left(S - M_{B_H}^2\right)^2 + M_{B_H}^2 \Gamma_{B_H}^2} \end{aligned}$$

$$
\begin{aligned}
&+\left[\left(g_V^{B_H ee}\right)^2+\left(g_A^{B_H ee}\right)^2\right] \\
&\times\left[4\left(1-\frac{\beta^2}{3}\right)\left[\left(g_V^{B_H tt}\right)^2+\left(g_A^{B_H tt}\right)^2\right]\right. \\
&\quad\left.-4\left(1-\beta^2\right)\left(g_A^{B_H tt}\right)^2\right]\frac{S^2}{\left(S-M_{B_H}^2\right)^2+M_{B_H}^2\Gamma_{B_H}^2} \\
&+\left(g_V^{Zee}g_V^{B_H ee}+g_A^{Zee}g_A^{B_H ee}\right) \\
&\times\left[4\left(1-\frac{\beta^2}{3}\right)\left(g_V^{Ztt}g_V^{B_H tt}+g_A^{Ztt}g_A^{B_H tt}\right)\right. \\
&\quad\left.-4\left(1-\beta^2\right)\left(g_A^{B_H tt}\right)\left(g_A^{Ztt}\right)\right] \qquad (13)\\
&\left.\times\frac{2S^2\left[\left(S-M_Z^2\right)\left(S-M_{B_H}^2\right)+M_Z\Gamma_Z M_{B_H}\Gamma_{B_H}\right]}{\left[\left(S-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right]\left[\left(S-M_{B_H}^2\right)^2+M_{B_H}^2\Gamma_{B_H}^2\right]}\right\},
\end{aligned}
$$

$$
\begin{aligned}
&M_2^{Z_H}(t\bar{t}) \\
&=\beta\left\{\frac{2e^2}{3}g_A^{Zee}g_A^{Ztt}\frac{4S\left(M_Z^2-S\right)}{\left(S-M_Z^2\right)^2+M_Z^2\Gamma_Z^2}\right. \\
&+\frac{2e^2}{3}g_A^{Z_H ee}g_A^{Z_H tt}\frac{4S\left(M_{Z_H}^2-S\right)}{\left(S-M_{Z_H}^2\right)^2+M_{Z_H}^2\Gamma_{Z_H}^2} \\
&+g_V^{Zee}g_A^{Zee}g_V^{Ztt}g_A^{Ztt}\frac{8S^2}{\left(S-M_Z^2\right)^2+M_Z^2\Gamma_Z^2} \qquad (14)\\
&+g_V^{Z_H ee}g_A^{Z_H ee}g_V^{Z_H tt}g_A^{Z_H tt}\frac{8S^2}{\left(S-M_{Z_H}^2\right)^2+M_{Z_H}^2\Gamma_{Z_H}^2} \\
&+\left(g_V^{Zee}g_A^{Z_H ee}+g_V^{Z_H ee}g_A^{Zee}\right)\left(g_V^{Ztt}g_A^{Z_H tt}+g_A^{Ztt}g_V^{Z_H tt}\right) \\
&\left.\times\frac{4S^2\left[\left(S-M_Z^2\right)\left(S-M_{Z_H}^2\right)+M_Z\Gamma_Z M_{Z_H}\Gamma_{Z_H}\right]}{\left[\left(S-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right]\left[\left(S-M_{Z_H}^2\right)^2+M_{Z_H}^2\Gamma_{Z_H}^2\right]}\right\},
\end{aligned}
$$

$$
\begin{aligned}
&M_1^{Z_H}(t\bar{t}) \\
&=\left\{\frac{16e^4}{9}\left(1-\frac{\beta^2}{3}\right)\right. \\
&+\frac{8e^2}{3}\left(1-\frac{\beta^2}{3}\right)g_V^{Zee}g_V^{Ztt}\frac{2S\left(M_Z^2-S\right)}{\left(S-M_Z^2\right)^2+M_Z^2\Gamma_Z^2} \\
&\times\left[\left(g_V^{Zee}\right)^2+\left(g_A^{Zee}\right)^2\right] \\
&\times\left[4\left(1-\frac{\beta^2}{3}\right)\left[\left(g_V^{Ztt}\right)^2+\left(g_A^{Ztt}\right)^2\right]\right. \\
&\quad\left.-4\left(1-\beta^2\right)\left(g_A^{Ztt}\right)^2\right]\frac{S^2}{\left(S-M_Z^2\right)^2+M_Z^2\Gamma_Z^2} \\
&+\frac{8e^2}{3}\left(1-\frac{\beta^2}{3}\right)g_V^{Z_H ee}g_V^{Z_H tt}\frac{2S\left(M_{Z_H}^2-S\right)}{\left(S-M_{Z_H}^2\right)^2+M_{Z_H}^2\Gamma_{Z_H}^2} \\
&+\left[\left(g_V^{Z_H ee}\right)^2+\left(g_A^{Z_H ee}\right)^2\right] \\
&\times\left[4\left(1-\frac{\beta^2}{3}\right)\left[\left(g_V^{Z_H tt}\right)^2+\left(g_A^{Z_H tt}\right)^2\right]\right. \\
&\quad\left.-4\left(1-\beta^2\right)\left(g_A^{Z_H tt}\right)^2\right]\frac{S^2}{\left(S-M_{Z_H}^2\right)^2+M_{Z_H}^2\Gamma_{Z_H}^2} \\
&+\left(g_V^{Zee}g_V^{Z_H ee}+g_A^{Zee}g_A^{Z_H ee}\right) \\
&\times\left[4\left(1-\frac{\beta^2}{3}\right)\left(g_V^{Ztt}g_V^{Z_H tt}+g_A^{Ztt}g_A^{Z_H tt}\right)\right. \\
&\quad\left.-4\left(1-\beta^2\right)\left(g_A^{Z_H tt}\right)\left(g_A^{Ztt}\right)\right] \qquad (15)\\
&\left.\times\frac{2S^2\left[\left(S-M_Z^2\right)\left(S-M_{Z_H}^2\right)+M_Z\Gamma_Z M_{Z_H}\Gamma_{Z_H}\right]}{\left[\left(S-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right]\left[\left(S-M_{Z_H}^2\right)^2+M_{Z_H}^2\Gamma_{Z_H}^2\right]}\right\}.
\end{aligned}
$$

In the above equations, we have assumed that the initial electron and positron beams are not polarized.

To see whether the new gauge bosons B_H and Z_H can produce significant deviations from the SM prediction value for $A_{\mathrm{FB}}(t\bar{t})$, we plot the relative correction parameters $R'_{B_H} = \delta A_{\mathrm{FB}}^{B_H}(t\bar{t})/A_{\mathrm{FB}}^{\mathrm{SM}}(t\bar{t})$ and $R'_{Z_H} = \delta A_{\mathrm{FB}}^{Z_H}(t\bar{t})/A_{\mathrm{FB}}^{\mathrm{SM}}(t\bar{t})$ as functions of M_{B_H} and M_{Z_H} in Figs. 5 and 6, respectively. From Figs. 5 and 6 we can see that in most of the parameter space preferred by the electroweak precision data, the absolute values of the relative correction parameters R'_{B_H} and R'_{Z_H} are smaller than 5%. The absolute values of R'_{Z_H} is larger than 5% only for the mixing parameter $c = 0.5$ and $1\,\mathrm{TeV} \leq M_{Z_H} \leq 1.4\,\mathrm{TeV}$. B_H exchange makes the deviation of the forward–backward asymmetry $A_{F_{\mathrm{B}}}(t\bar{t})$ from its SM value may be positive or negative, which depends on the B_H mass M_{B_H}. The resonance peak can emerge for $M_{B_H} \approx 800\,\mathrm{GeV}$. Furthermore, the absolute value of R'_{B_H} increases as the mixing parameters c' and x_{L} are increasing. For $c' \geq 0.71$, $x_{\mathrm{L}} \geq 0.5$, and $600\,\mathrm{GeV} \leq M_{B_H} \leq 1000\,\mathrm{GeV}$, the absolute value of R'_{B_H} is larger than 5%, which might be detected in the future LC experiments. However, for $c' \leq 0.68$ and $x_{\mathrm{L}} \leq 0.4$, except for a small region near $M_{B_H} = 800\,\mathrm{GeV}$, the absolute value of R'_{B_H} is smaller than 5%.

Similar to the above calculation, we can obtain the corrections of B_H exchange and Z_H exchange to the forward–backward symmetry $A_{\mathrm{FB}}(f\bar{f})$ with $f = \mu, \tau, b$ or c. From the coupling formula of the new gauge bosons B_H and Z_H to differently fermions given in [7], we can surmise that the conclusions are similar to those for $A_{\mathrm{FB}}(t\bar{t})$. We have confirmed this expectation through explicit calculation. Certainly, the contributions of B_H exchange to $A_{\mathrm{FB}}(f\bar{f})$ mainly are dependent on the free parameters M_{B_H} and c', while the contributions of B_H exchange to $A_{\mathrm{FB}}(t\bar{t})$ mainly depend on the free parameters M_{B_H}, c', and x_{L}.

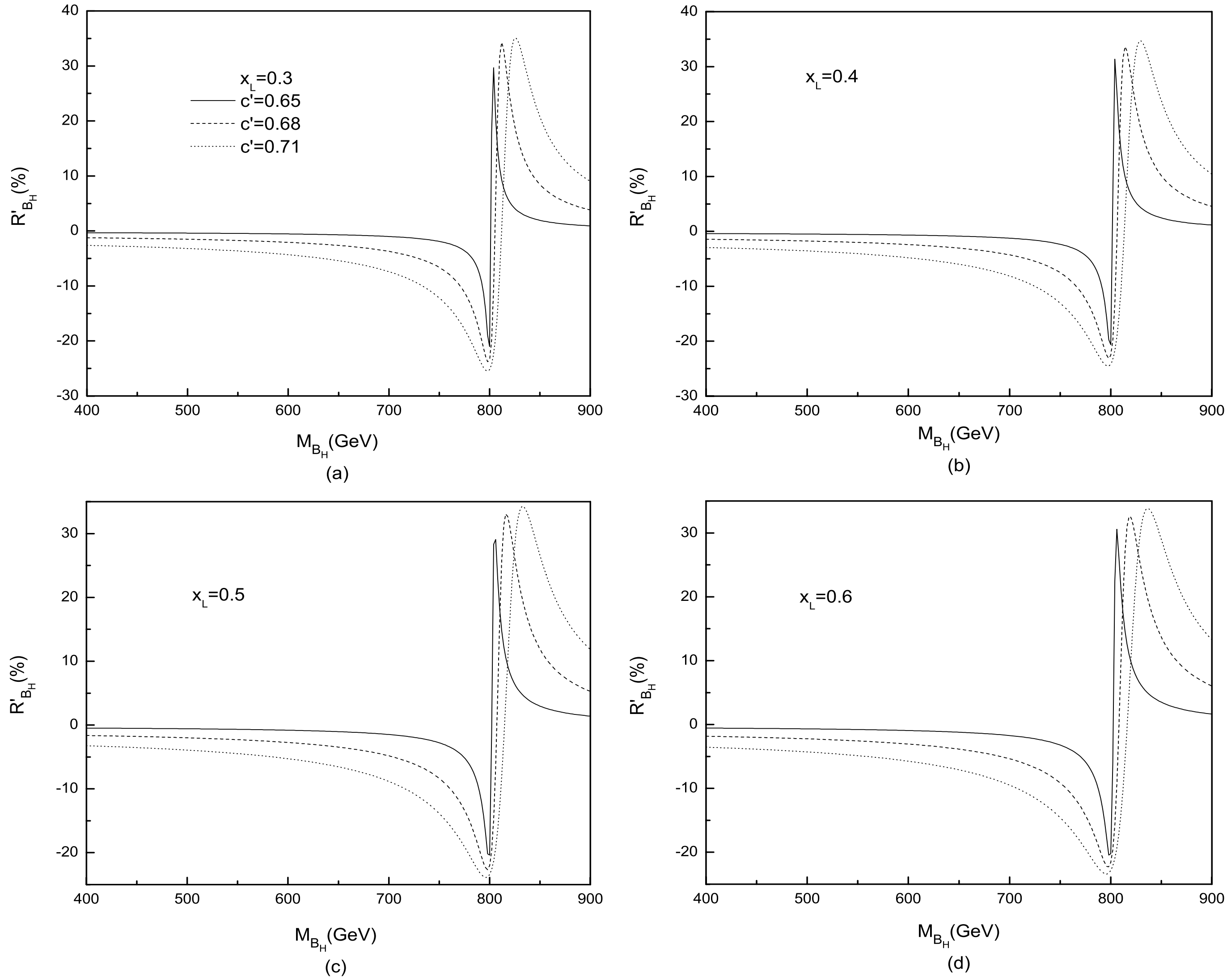

Fig. 5a–d. The relative correction parameter R'_{B_H} as a function of M_{B_H} for different values of the mixing parameters c' and x_L

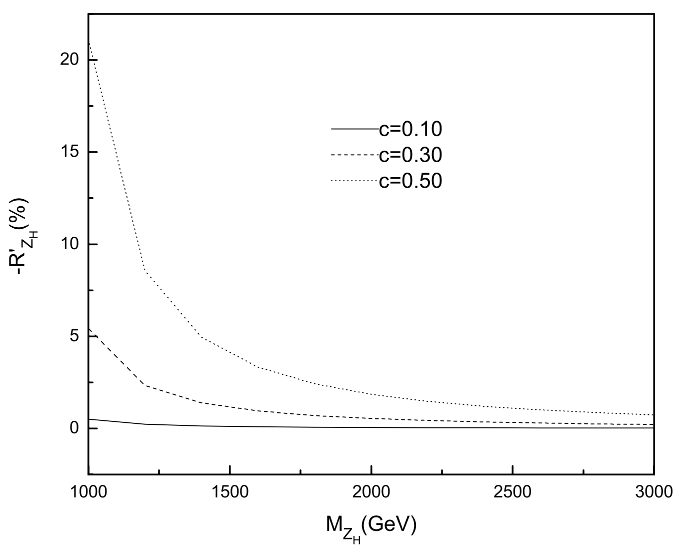

Fig. 6. The relative correction parameter R'_{Z_H} as a function of M_{Z_H} for three values of the mixing parameter c

5 Conclusions and discussions

An LC will be an ideal machine for precisely testing the SM and probing NP beyond the SM. Some kinds of NP predict the existence of new particles that will be manifest as a rather spectacular resonance in the LC experiments if the achievable CM energy $\sqrt{S}$ is sufficient. Even if their masses exceed the CM energy $\sqrt{S}$, the LC experiments also retain an indirect sensitivity through a precision study of their virtual corrections to observables.

It is widely believed that the top quark, with a mass of the order of the electroweak scale, will be a sensitive probe into NP beyond the SM. The quantum correction effects of the new particles to some SM processes involving the top quark are more important than those for lighter fermions. Thus, the top quark plays a key role in the quest for deviations of observables from their SM predictions. On the other hand, top quark pairs can be copiously produced mainly through the process $e^+e^- \to t\bar{t}$ in the future LC experiments. So, in this paper we discuss and calculate the corrections of the new gauge bosons B_H and Z_H predicted by the LH model to the production cross section $\sigma(t\bar{t})$ and the forward–backward asymmetry $A_{\mathrm{FB}}(t\bar{t})$ of the process $e^+e^- \to t\bar{t}$.

The LH model has all essential features of the little Higgs models. So, in this paper we give our numerical results in the context of the LH model, although many alternatives have been proposed [2, 3]. We find that the new gauge bosons Z_H and B_H can produce significant correction effects on the process $e^+e^- \to t\bar{t}$, which can be further enhanced by suitably polarized beams. In most of the parameter space $f = 1\,\mathrm{TeV} \sim 2\,\mathrm{TeV}, c' = 0.62 \sim 0.73, c = 0.1 \sim 0.5$, and $x_{\mathrm{L}} = 0.3 \sim 0.6$, consistent with the electroweak precision data, and the absolute value of the relative correction parameter R_{B_H} generated by B_H exchange is larger than 5%. As long as $1\,\mathrm{TeV} \leq M_{Z_H} \leq 1.5\,\mathrm{TeV}$ and $0.3 \leq c \leq 0.5$,, the absolute value of R_{Z_H} is larger than 5%. Thus, we can say that, with reasonable values of the parameters in the LH model, the possible signals of the new gauge bosons B_H and Z_H can be detected via the process $e^+e^- \to t\bar{t}$ in the future LC experiments with the CM energy $\sqrt{S} = 800\,\mathrm{GeV}$. However, B_H exchange and Z_H exchange can only generate very small corrections to the forward–backward asymmetry $A_{\mathrm{FB}}(t\bar{t})$ in most of the parameter space. It is possible that, in a very small range of the parameter space, the possible signals of B_H and Z_H might be detected via measuring the deviations of $A_{\mathrm{FB}}(t\bar{t})$ from its SM prediction.

The couplings of the new gauge boson B_H to fermions are quite model dependent, which depend on the choice of the fermion $U(1)$ charges under the two $U(1)$ groups. The $U(1)$ charges of the SM fermions are constrained by requiring that the Yukawa couplings are gauge invariant and maintaining the usual SM hypercharge assignment. Combing the gauge invariance of the Yukawa couplings with the $U(1)$ anomaly-free can fix all of the $U(1)$ charge values. In this paper, we have used the couplings of the B_H to fermions, which come from this kind of choice. Certainly, this is only one example of all possible $U(1)$ charge assignments. In other little Higgs models, several alternatives for the $U(1)$ charge choice exist [2, 3, 10], and the numerical results for the new gauge boson B_H obtained in this paper might be changed.

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